

WOO! AESTHETIC VARIATIONS OF THE “SAME” LESSON

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Efforts to enhance the aesthetic impact of mathematics lessons must account for the role of teachers in shaping the unfolding mathematical content of their enacted lessons. In this paper, we draw from Dietiker (2015) to describe differences in the mathematical stories of the enacted lessons of two veteran teachers teaching the same lesson. We identify connections between these differences and the variations in student experiences as illustrated by visible student reactions.

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How a story is told matters. For example, learning at the beginning of the play that Romeo and Juliet will die (e.g., in Shakespeare’s play) offers an audience a very different aesthetic experience than when the storyteller chooses to withhold this information (e.g., in the 1968 movie). When mathematics lessons are interpreted as stories (Dietiker, 2013, 2015), the same dynamic can be recognized. The way in which information in a mathematical story is withheld and revealed can significantly impact the aesthetic experience of the students.

The aesthetic nature of mathematics teaching and learning has been largely ignored and needs more study. Although research has begun to understand the aesthetic dimensions of mathematical learning (e.g., Sinclair, 2004), little is known about how mathematics lessons can be crafted to take advantage of aesthetic opportunities. We use the term “craft” to describe the role of both designing curriculum and of teaching. Interpreting teaching as creative work is consistent with Brown (2009), who suggests that teaching with written curricula is similar to how musicians perform with sheet music. He argues that, “practitioners bring to life the composer’s initial concept through a process of interpretation and adaptation... In both cases, no two renditions of practice are exactly alike” (Brown, 2009, p. 17).

In this paper, we demonstrate that in a mathematics classroom, how a story is told is just as important to its aesthetic impact as the story itself. That is, if we want to create opportunities for surprise or anticipation, we need to do more than design new mathematics curriculum and instead must consider the ways teachers enable and constrain how the content unfolds throughout the lesson. We present a case study of two veteran teachers teaching the same lesson from the same textbook with notably different student reactions to demonstrate the aesthetic variations of telling the “same” mathematical story. We also explain how these aesthetic differences resulted from the way in which the mathematical content unfolded throughout the lesson enactments.

Theoretical Framework

Mathematical lessons can be interpreted as mathematical stories (Dietiker, 2015). This interpretation specifically offers a way to recognize the logical and aesthetic dimensions of the unfolding mathematical content of a lesson. Not limited to contextual story problems, this framing foregrounds sequential changes that occur over time (“acts”), defined as the distinct portions of the mathematical lesson which can be identified by changes in the mathematical characters (the objects of the lesson, such as a quadratic function), actions (the acts by a student or teacher to manipulate an object), and/or settings (the representational “space” in which the mathematical characters and

actions are found). Just as literary stories are “told” by a narrator, mathematical stories are narrated by the utterances of the teachers and students.

In order to recognize the aesthetic dimensions of a mathematical story, we focus on the mathematical plot, which in literature is the tension felt by students between what is known and what is desired to be known in the story (Nodelman & Reimer, 2003). Analyzing the mathematical plot enables the investigation of how a mathematical sequence can generate suspense and surprise. Barthes’ (1974) describes the transition from question to answer with codes: *question formulation*, *promise of an answer*, *snare* (misleading direction), *equivocation* (misleading ambiguity), *jamming* (the question is unanswerable), *suspended answer* (the delay of the answer), *partial answer* (progress), and *disclosure of the answer* (endorsing the answer). The transition from asking to answering a question forms a story arc. In mathematics classrooms, story arcs include questions asked by teachers, students, and curriculum. In addition, some story arcs may be based on implicit questions, raised by the goals of an activity but never stated.

Just as is the case with literary stories, the aesthetic value of a mathematical story varies by individual. That is, there is no “best” way a story can be told for everyone. However, literary theory suggests that there is a relationship between the form and function of narrative. It is thus an assumption of the mathematical story framework that distinguishing the different forms of the unfolding mathematical content throughout a lesson can identify how observed aesthetic reactions of students can be understood. For example, long story arcs containing nested shorter story arcs may give a larger purpose to the shorter questions and, thus, coherence for the lesson. In fact, without long story arcs, a long sequence of short story arcs may prevent a student/reader from recognizing a purpose for each question. Longer story arcs may also create opportunities for twists in the plot that lead to surprise and anticipation. In this study, we examine the relationship between the form of the mathematical plot and the evident aesthetic of an enacted lesson by asking the question, “How can enactments of the same written curriculum differ and what role do these differences play in the aesthetic reactions of the students?”

Methods

The current study compares two enactments by different algebra teachers based on the same textbook lesson. The goal of this research project is to learn more about the different ways that expert teachers, from diverse settings and communities, enact the “same” curriculum lessons in their respective classrooms. The lesson selected for this analysis focused on a method of solving quadratic equations by factoring using the zero product property.

In order to minimize the interference of classroom management issues on curriculum enactment, or unfamiliarity with the same written curriculum (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2012), the teacher participants were required to have at least five years experience teaching mathematics, at least three years of which must have involved the selected written curriculum. Additionally, participants were selected only if they had a strong record of excellence in teaching (e.g. National Board certification, receiving a teaching award, a regular leader of professional development, etc.). The selected enactments were taught by two teachers in two different geographical regions (Mr. J and Ms. W) with 8 and 20 years of teaching, respectively, and 4 and 10 years experience, respectively, using the textbook.

The enactments were observed and videotaped in Spring, 2015, which included the video recording of the whole class as well as one focal student group within each class. Since a portion of each enacted lesson contained group work, the mathematical plots described in this paper represent the mathematical stories from the perspective of this selected group of students, including both whole class discussion and small group discourse.

To interpret the mathematical plots, the videos were transcribed and then analyzed to identify where acts start and stop by identifying when the mathematical characters, actions, and/or settings

changed and the mathematical story advanced. Next, the researchers identified all of the mathematical questions formulated throughout the lesson, identifying questions that were asked by the teacher, student, or curriculum. Finally, each of the acts were coded using Barthes' codes (described earlier) by researchers in pairs who then came together as a whole group to resolve differences. The resulting mathematical plot diagrams for the two enactments are displayed in Figure 1 (for Ms. W's lesson) and Figure 2 (for Mr. J's lesson). In these diagrams, a shaded cell without a code means the question is still open. Note that the colors were used to highlight formulated questions that were common to both lessons. In addition, the width of the columns containing the acts do not signify increments of time. Rather, the columns represent the acts, which are portions of the lesson for which the story changes. These acts vary in elapsed time, which can be seen in the elapsed times provided in these diagrams.

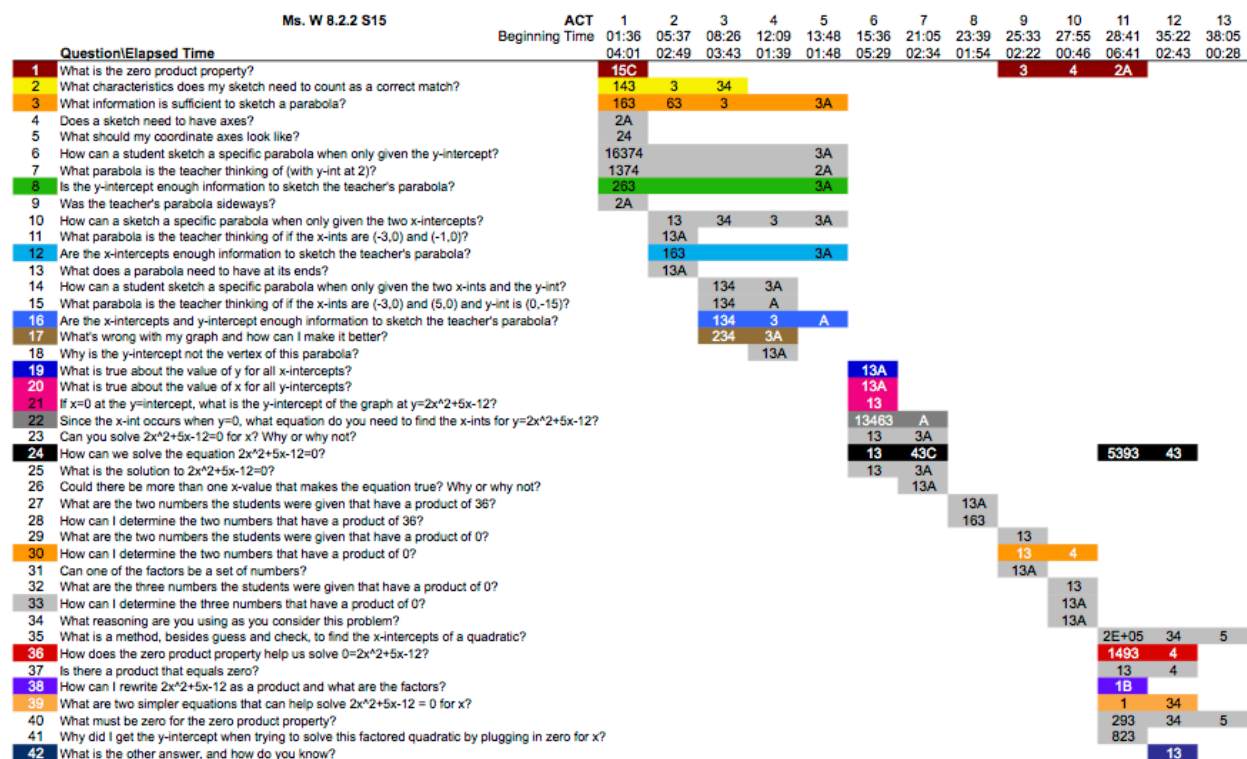


Figure 1. Ms. W's mathematical plot, where 1: formulated question by teacher, researcher, or textbook, 2: formulated question by student, 3: progress by student, 4: progress by teacher or environment, 5: promise, 6: equivocation, 7: jamming, 8: proposal, 9: snare by student, A: disclosure by teacher, B: disclosure by student, C: suspended answer. Colors other than light grey indicate a question that was identified in both enactments.

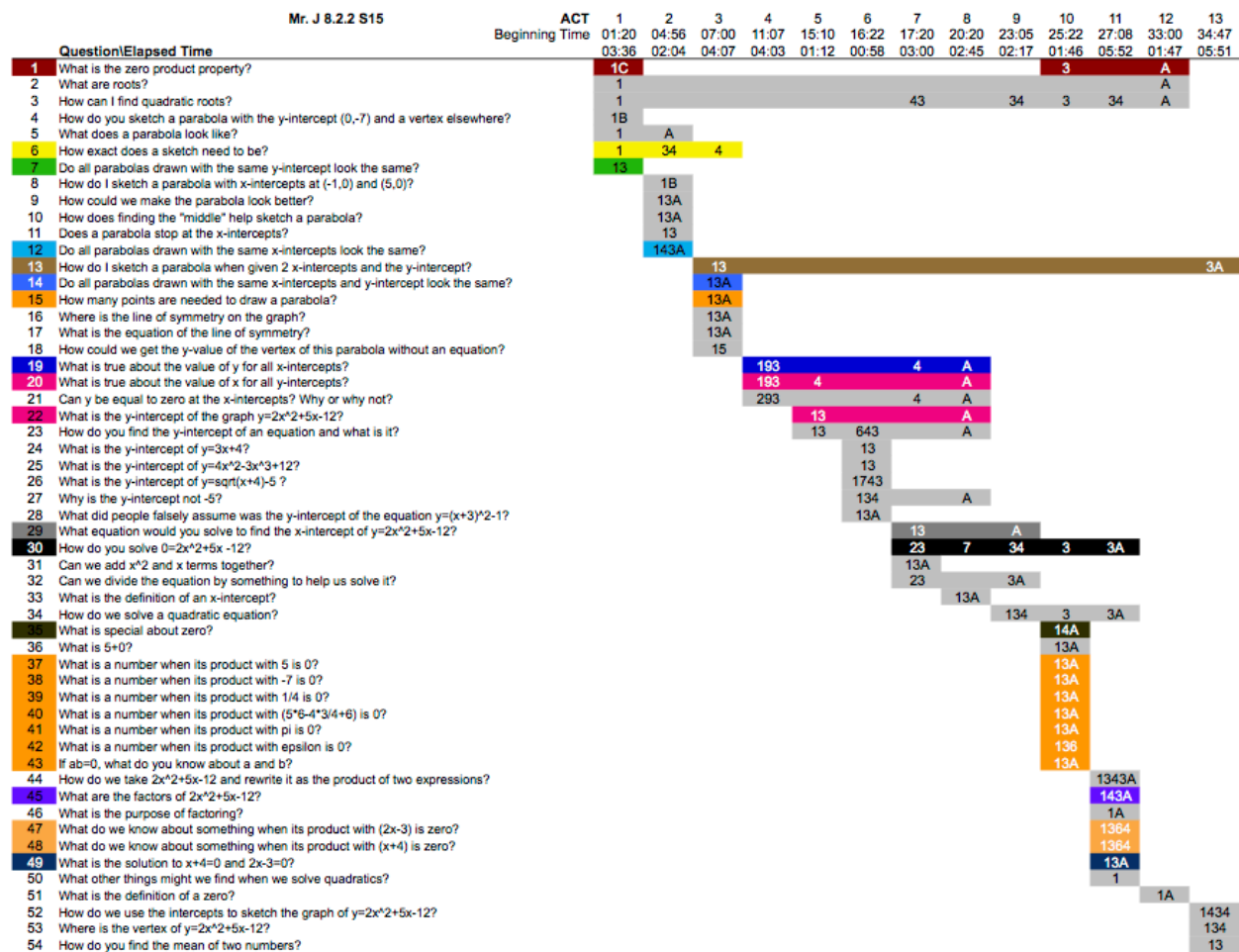


Figure 2. Mr. J's mathematical plot. See the caption in Figure 1 for the coding reference. Colors other than light grey indicate a question that was identified in both enactments.

Findings

On the surface, these lesson enactments appear very similar. Both enactments contained 13 acts and were approximately the same length (Mr. J: 39 minutes, Ms. W: 37 minutes). Both teachers focused their lessons on the same set of tasks in the algebra textbook, omitting the same task at the end of the lesson. Sixteen mathematical questions were common to both lessons and, with the exception of two questions, these were all introduced in the same order. A structural analysis of these diagrams can be found in Richman, Dietiker, and Brakoniecki (2016).

Despite these similarities, interesting differences in how the mathematical ideas unfolded across the lessons are evident. For example, Mr. J's lesson contained 54 story arcs while Ms. W's had 42. Ms. W's enactment contained a higher proportion of story arcs that remained open for multiple acts (52% compared to 30%) and yet her story arcs collectively demonstrate that her lesson contained two separate and disjointed activities while Mr. J's lesson had story arcs that unified the two activities. In addition, Mr. J's enactment had more story arcs open per act (an average of 9.4 story arcs open per act as compared to Ms. W's 6.8).

At this top level, it may be difficult to recognize how and why particular moments of a lesson occurred. Thus, comparisons of the two activities are next described to explain potential aesthetic differences for students in these two enactments.

Determining a Parabola

Both teachers began with the first task of the textbook lesson. The task prompted students to consider the number of points needed to determine a parabola. In both enactments, the same question (*What information is sufficient to sketch a parabola?*) arose, yet the way in which this question played out in the two classrooms was different. Ms. W's enactment enabled the question to arise early and remain open for Acts 1 through 5. In contrast, this question did not emerge in Mr. J's enactment until Act 3, after which it was answered quickly (see Figure 3). As with the plot diagrams in Figures 1 and 2, the width of each cell does not indicate the amount of time elapsed for the given act. Instead, the diagram is formatted to have the same overall width so that Acts 1-5 of Ms. W's lesson represents the same activity as Acts 1-3 of Mr. J's lesson.

What information is sufficient to sketch a parabola?					
Ms. W's Lesson	1	2	3	4	5
Question #3	163	63	3		3A

Mr. J's Lesson	1	2	3
Question #15			13A

Figure 3. Story arcs representing how the question “What information is sufficient to sketch a parabola?” was raised and answered in two different enactments. See the caption of Figure 1 for the reference of numerical codes.

Ms. W. In Act 1, Ms. W revealed the y -intercept of an unknown parabola and asked students to sketch it. When students encountered difficulty, “What information is sufficient to sketch a parabola?” was implicitly raised (*formulated question*, code 1). The ambiguity on whether the information given was sufficient, was evident by students, “Wait, that’s the only clue we get?” (*equivocation*, code 6). Another student notes, “It could be so many things!” (*progress*, code 3). Next, [Act 2] the teacher challenged the students to sketch a second parabola with only the x -intercepts (*equivocation*, code 6). Anticipation was evident in the celebration by two students when they guessed correctly with high fives and “Woo!” (*progress*, code 3). During the third challenge [Act 3], a student described his struggle to sketch a parabola with symmetry (*progress*, code 3). In Act 4, Ms. W asked this student to recount his progress to the class (*no new information*). In a discussion [Act 5], a student noted that with only one or two points the parabola could point up or down (*progress*, code 3). The answer to “What information is sufficient to sketch a parabola?” was finally disclosed when, in response to a student question, Ms. W revealed her parabolas (*disclosure*, code A). Ms. W then explained that the students who guessed the second parabola got lucky since the exact graph could only be determined with three points.

Mr. J. Mr. J prompted students to sketch a parabola when given the y -intercept [Act 1] and then asked students to compare with a peer. He repeated this process with two x -intercepts, and then x - and y -intercepts [Acts 2 and 3, respectively]. With this framing, the question of whether there was enough information to sketch a particular parabola was not even raised (which is why there are no codes for this story arc during Acts 1 and 2). Instead, the driving questions were “what parabola passes through ____?” and “are the parabolas the same?” As a consequence, the question of how many points are necessary to determine a parabola was not raised until the end of the activity, in Act 3, when Mr. J said “So, how many points are needed to draw a parabola?” (*formulated question*, code 1) to which the students responded “3” (*progress*, code 3). The teacher endorsed this response, disclosing the question (*disclosure*, code A).

Identifying the Roots of a Parabola

Next, both teachers prompted student groups to work on the same task, asking students how they could use what they know about intercepts (e.g., for the y -intercept, $x = 0$) to solve a quadratic equation for its roots. In both enactments, the same question based on this task arose, yet its resolution was again strikingly different (see Figure 4). In Ms. W's enactment, the question was considered in Acts 6 and 7, was interrupted by another activity, and then re-appeared in Acts 11 and 12. In contrast, in Mr. J's enactment, the question was raised in Act 7 and remained open until Act 11. Even more striking, however, were the differences in the way in which the question was answered over the course of this portion of the lesson, as is evident by the differences in the types and distribution of codes. These differences are described below.

How can we solve the equation $2x^2 + 5x - 12 = 0$?								
Ms. W's Lesson	6	7	8	9	10	11	12	
Question #24	13	43C				5393	43	

Mr. J's Lesson	4	5	6	7	8	9	10	11	12
Question #30				23	7	34	3	3A	

Figure 4. Story arcs representing how the question “How can we solve the equation $2x^2 + 5x - 12 = 0$?” was raised and answered in two different enactments. See the caption of Figure 1 for the reference of numerical codes.

Ms. W. At the start of Act 6, Ms. W assigned the task (*formulated question*, code 1) and students worked in groups while the teacher circulated (*progress*, code 3). The focal group began to consider what values would make $2x^2 + 5x$ equal to 12. Ms. W called the class back together [Act 7] and indicated that students should be trying to solve $0 = 2x^2 + 5x - 12$ (*progress*, code 4). A student from the focal group shared that they were able to determine solutions $x = -4$ and $x = 1.5$ by guess and check (*progress*, code 3). Ms. W then shifted their attention (*suspended answer*, code C) to a game that appeared unrelated at the time [Acts 8, 9, and 10]. During this, she assigned volunteers “secret” numbers and asked them to reveal the product of the numbers. With the product, the rest of class had to guess what the secret numbers were. Through this activity, the students realized that when the product of two or more numbers is zero, at least one of the numbers must be zero. Ms. W then [Act 11] had her students return to the task, indicating that this property can help find the solution to $0 = 2x^2 + 5x - 12$ (*promise*, code 5). The focal group struggled; after factoring the expression (*progress*, code 3), they substituted 0 for x in the factored expression (*snare*, code 9). One student gasped audibly and exclaimed “Oh my god, guys!” upon recognizing that doing this yields the number -12, which is the y -intercept of the equation. The group then realized that the x -intercepts occur when y equals zero, (*progress*, code 3) and shifted to solving $0 = 2x^2 + 5x - 12$. They knew that something had to be done with a factored form, so they tried a mixture of manipulations to rewrite the quadratic expression to no avail. The teacher arrived at the group and the students asked for help [Act 12]. A student said she got the answer $x = 1.5$ by guessing. The teacher then reminded the students of the game from Acts 8, 9, and 10 (*progress*, code 4), after which one of the students connected this answer to the solution of $2x - 3 = 0$ (*progress*, code 3).

Mr. J. Mr. J began this activity by focusing students on the definitions [Acts 4 and 5] and misconceptions [Act 6] regarding x - and y -intercepts. Groups then collaborated on solving $0 = 2x^2 + 5x - 12$, asking each other questions about strategy [*formulated question* and *progress*, codes 2 and 3, Act 7]. Mr. J again called the students back together and indicated that this task should

be challenging, indicating perhaps it is unsolvable [*jamming*, code 7, Act 8]. He then reviewed the definitions of x - and y -intercepts and to discuss how to find the y -intercept of $y = 2x^2 + 5x - 12$. Next [Act 9], he asked students to try different algebraic strategies in an attempt to solve $0 = 2x^2 + 5x - 12$ (*progress*, code 3). When students struggled, Mr. J had students focus on how strategies that work for other equations do not work for quadratics (*progress*, code 4). In Act 10, Mr. J displayed a sheet of missing factor multiplication problems whose products are all 0. Students responded that the missing factor had to be zero in these problems (*progress*, code 3). Mr. J returned the class to $0 = 2x^2 + 5x - 12$ and reviewed with the class how to factor the expression to be the product of two binomials $(x + 4)(2x - 3)$, and then use the zero product property to figure out the roots. He then reveals the answers to be $(-4, 0)$ or $(1.5, 0)$ [*disclosure*, code A, Act 11]. Mr. J then had his students copy definitions into their notebooks for roots, zeros, x -intercepts, and the zero product property [no new progress, Act 12].

Differences in Storytelling

Collectively, these two episodes, which were based closely on the same written tasks, demonstrate that teachers influence the way that mathematical content unfolds throughout their lessons. Together with the students, the teachers craft a mathematical story that has aesthetic dimensions. Despite having so many similarities at a lesson level, the structures of the plot diagrams of these two mathematical stories are strikingly different.

As described in this paper, Ms. W's framing of the parabola sketching activity as a prolonged challenge led to several important aesthetic and mathematical differences. The question "What information is sufficient to sketch a parabola?" was introduced at the start and, as a result, this question was the driving mathematical focus of the activity. That is, the challenge to figure out the teacher's parabola gave purpose to the mathematical inquiry. Thus, the mathematical story in Ms. W's class included mathematical revelations of why only having the y -intercept or only having the two x -intercepts are insufficient. That different parabolas could result when given insufficient information was part of Mr. J's lesson, however, why that is the case was not. In the aesthetic dimensions of these enactments, Ms. W's framing enabled the students to be surprised by the lack of information and curious about the result (by later asking for the answer). This approach engaged students and resulted in visible celebration. In contrast, since Mr. J did not pose this question until after the parabola sketching activity, his students (while attentive and cooperative through the activity) barely participated in addressing the question of the number of points that are necessary to determine a parabola when finally asked.

When looking at the plot diagrams for the question "How can we solve the equation $0 = 2x^2 + 5x - 12$?" (Figure 4), we see a different structure than the plot diagrams during the parabola sketching activity (Figure 3). In Ms. W's classroom, the question was under consideration until it became temporarily suspended when attention was shifted to a seemingly unrelated game. Upon completion of this exploration, Ms. W's released her class to reconsider the question again, but now taking into consideration some takeaways from the zero product property game. It was through this open exploration that the students in the group experimented with different strategies and were able to have a visible aesthetic reaction to finding out that substituting in zero for x in the factored form of an equation still yields the y -intercept of the equation. In contrast, in Mr. J's classroom, the development of the mathematical ideas related to solving this question primarily flowed through Mr. J. He focused students' attention toward the problem for exploration, recapped what they already knew and don't know, helped discover a relevant property, and showed how this property could be used to solve the original problem. When this question was under consideration, the students in Ms. W's classroom were what could be called the mathematical actors, while in Mr. J's classroom, the

teacher was the leading actor of the mathematical story. This could explain the students' aesthetic reaction to their mathematical discovery in Ms. W's classroom, and the apparent absence of one in Mr. J's classroom.

While the analyses of the plot diagrams of these individual questions help highlight possible influences on the aesthetic reactions and opportunities for students, analyses of the plot diagrams of the entire lesson yield other insights. Just as Ms. W's overarching question of "What information is sufficient to sketch a parabola?" offered coherence and purpose to the parabola sketching activity, Mr. J's whole mathematical story included multiple overarching questions that together gave purpose and meaning for the entire lesson (particularly "How can I find the roots of a parabola" and "How can I sketch a parabola with the x -intercepts and y -intercept?"). Although there was no visible aesthetic effect to this crafting of the content, we suspect that there was coherence by the end of the lesson that enabled students to recognize that both episodes together were needed to achieve the lesson goal; namely, to sketch a parabola by its intercepts.

Discussion

If mathematics educators want to change the aesthetic nature of mathematics classrooms, can we just change the curriculum? Our answer is no. As shown in this study, the curriculum is not the only factor that determines the aesthetic impact of the enactment. When mathematics lessons are interpreted as mathematical stories and are compared in this way, differences in the experiences of students can not only be noticed but also understood. This recognition enables future research to focus on the storytelling; that is, to explore how mathematics teachers can craft lessons that can offer engaging learning experiences for students.

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